

3.2 Solve Linear Systems Algebraically

Substitution Method	Elimination Method
<ol style="list-style-type: none"> <li>Solve one equation for one variable. (look for a coefficient of 1)</li> <li>Substitute this expression into the OTHER equation.</li> <li>Solve for the remaining variable.</li> <li>Substitute this value into EITHER equation to solve for the remaining variable.</li> <li>State the final answer.                             <ul style="list-style-type: none"> <li>➤ Ordered pair (alphabetically)</li> <li>➤ Slope-intercept form</li> <li>➤ No Solution</li> </ul> </li> </ol> <p>★ look for a coefficient of 1 or -1 that is easy to solve for</p>	<ol style="list-style-type: none"> <li>Pick a variable to eliminate. <span style="float: right;">+3x -3x</span></li> <li>GOAL: to create opposite terms                             <ul style="list-style-type: none"> <li>➤ May only have to change one equation</li> <li>➤ May have to change both equations by multiplying each by the other's coefficient</li> </ul> </li> <li>ADD the 2 equations. One variable should be eliminated.</li> <li>Solve for the remaining variable.</li> <li>Substitute this value into EITHER equation to solve for the remaining variable.</li> <li>State the final answer.                             <ul style="list-style-type: none"> <li>➤ Ordered pair (alphabetically)</li> <li>➤ Slope-intercept form</li> <li>➤ No Solution</li> </ul> </li> </ol>

Substitution Examples:

<p>1. <math>3x - 2y = -3</math> solve for y  <math>3x + y = 3</math>  <math>y = 3 - 3x</math> substitute in for y  <math>3x - 2(3 - 3x) = -3</math>  <math>3x - 6 + 6x = -3</math>  <math>9x - 6 = -3</math>  <math>9x = 3</math>  <math>x = 1/3</math>                  plug in to other equation <math>y = 3 - 3(1/3)</math>  <math>y = 3 - 1</math>  <math>y = 2</math>  <math>(1/3, 2)</math></p>	<p>2. <math>3x - 2y = 10</math>  <math>x + 4y = -6</math>  <math>x = -6 - 4y</math>  <math>3(-6 - 4y) - 2y = 10</math>  <math>-18 - 12y - 2y = 10</math>  <math>-14y = 28</math>  <math>y = -2</math>  <math>x = -6 - 4(-2)</math>  <math>x = -6 + 8 = 2</math>  <math>(2, -2)</math></p>
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Write answer as ordered pair

### Elimination Examples:

$$\begin{cases} 2x + 3y = 2 \\ x - 3y = -17 \end{cases}$$

$$3x = -15$$

$$x = -5$$

$$-5 - 3y = -17$$

$$-3y = -12$$

$$y = 4$$

$$(-5, 4)$$

opposites  
so add  
equations

plugging  
to either  
equation  
to get  
y

$$\begin{cases} 5x + 3y = 0 \\ 4x + 5y = 13 \end{cases}$$

$$-20x - 12y = 0$$

$$20x + 25y = 65$$

$$13y = 65$$

$$y = 5$$

$$5x + 3(5) = 0$$

$$5x + 15 = 0$$

$$5x = -15$$

$$x = -3$$

$$(-3, 5)$$

multiply

### Special Cases:

$$5. \quad 2x + 6y = 16$$

$$\left(\frac{2}{3}x + 2y = 18\right)(3)$$

$$-2x - 6y = -54$$

$$\rightarrow 2x + 6y = 16$$

$$0 = -38$$

not true

so no solution

lines would be  
parallel!

$$6. \quad \left(\frac{1}{2}x - \frac{2}{3}y = 2\right)(6)$$

$$\left(\frac{4}{3}x - \frac{16}{9}y = \frac{16}{3}\right)(-9)$$

$$(3x - 4y = 12)(4)$$

$$-12x - 16y = 48$$

$$12x - 16y = 48$$

$$0 = 0$$

true

so lines coincide

answer is

$$3x - 4y = 12$$

$$\text{or } y = \frac{3}{4}x - 3$$

clear  
fractions  
first

Word Problems.

Example 1.

$x = \text{shorts cost}$      $y = \text{shirts cost}$

Morgan and Amy found a special on shorts and shirts at American Eagle. Morgan bought 5 pairs of shorts and 7 shirts for \$145. Amy bought 3 shorts and 6 shirts for \$105. Find the cost of each pair of shorts and each shirt.

$$\begin{array}{r} 5x + 7y = 145 \quad \cdot 3 \\ 3x + 6y = 105 \quad \cdot 5 \\ \hline -15x - 21y = -435 \\ 15x + 30y = 525 \\ \hline 9y = 90 \\ \boxed{y = 10} \end{array}$$

$$\begin{array}{r} 5x + 7(10) = 145 \\ 5x + 70 = 145 \\ 5x = 75 \\ \boxed{x = 15} \end{array}$$

shorts cost \$15  
shirts cost \$10

You try. The Bike shop owns three times as many mountain bikes as racing bikes. There are 60 more mountain bikes than racing bikes. How many of each does the company own?

$x = \text{mountain bikes}$      $y = \text{racing}$

$$\begin{array}{r} x = 3y \\ x = y + 60 \end{array}$$

$$3y = y + 60$$

$$\begin{array}{r} 2y = 60 \\ \boxed{y = 30} \end{array}$$

$$\boxed{x = 90}$$

90 mountain bikes  
30 racing bikes

Example 2. There were 166 people at the booster club bingo. The price was \$2 for adults and \$0.75 for students. The booster club raised \$293.25. How many adults and how many students attended?

$x = \# \text{ adults}$      $y = \# \text{ students}$

$$\begin{array}{r} x + y = 166 \quad \text{total people} \\ 2x + .75y = 293.25 \quad \text{total money} \end{array}$$

$$x = 166 - y$$

$$\begin{array}{r} 2(166 - y) + .75y = 293.25 \\ 332 - 2y + .75y = 293.25 \\ -1.25y = -38.75 \end{array}$$

$$y = 31$$

$$x = 166 - 31 = 135$$

135 adults  
31 students

